DUALITY VERSUS SUPERSYMMETRY AND COMPACTIFICATION

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Duality versus supersymmetry and compactification

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We study the interplay between T duality, compactification, and supersymmetry. We prove that when the original configuration has unbroken space-time supersymmetries, the dual configuration also does if a special condition is met: the Killing spinors of the original configuration have to be independent of the coordinate which corresponds to the isometry direction of the bosonic fields used for duality. Examples of "losers" (T duals are not supersymmetric) and "winners" (T duals are supersymmetric) are given.

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I. INTRODUCTION

Target-space duality (T duality) is a powerful tool for generating new classical solutions of string theory. It can be used in the σ model context to generate new exact solutions but also in the context of the leading order in α' effective action to generate new solutions to the low-energy equations of motion. Some of these solutions have unbroken supersymmetries. The purpose of this paper is to study the generic relation between the supersymmetric properties of the original configuration and the dual one in the context of the low-energy effective action.

It has been observed that in some cases T duality preserves unbroken supersymmetry. Well-known examples are the supersymmetric string wave solutions (SSW's) [1] and their partners, dual waves [2], which in particular include fundamental string solutions [3]. Another example of T-dual partners with unbroken supersymmetries is given by a special class of fivebrane solutions [4], multimonopoles [5] and their duals, a special class of stringy asymptotically locally Euclidean (ALE) instantons [6] which has a multicenter metric.

The preservation of unbroken supersymmetries by duality is related in principle to the fact that T duality is just one of the *hidden symmetries* of the supergravity theory that arises after dimensional reduction [7]. These hidden symmetries are indeed consistent with the supersymmetry of the dimensionally reduced theory. However, some recently discovered counterexamples seem to contradict this preliminary understanding. Therefore, one of the main goals of our analysis is to find the general condition that guarantees the preservation of unbroken supersymmetries that it is not satisfied by these counterexamples. We will perform this analysis in the context of $N = 1$, $d = 10$ supergravity without vector fields. More general results involving Abelian and non-Abelian vector fields and higher order α' corrections will be reported elsewhere [8]. Some of the results presented in this paper were announced in [9].

The first counterexample known to us appears in a very simple case. We have found some time ago[1] that if one starts with ten-dimensional flat space (which has all supersymmetries unbroken) in polar coordinates and performs a T-duality transformation with respect to the angular coordinate φ, the resulting configuration has no unbroken supersymmetries whatsoever.

The explanation of this apparently inconsistent situation will be found in a Kaluza-Klein-type analysis of the fermionic supersymmetry transformation rules of $N = 1$, $d = 10$ supergravity. In the conventional dimensional reduction of this theory by compactification of one dimension (with coordinate x, say) one only considers those field configurations that do not depend on the compact coordinate, and one only considers those supersymmetry transformations generated by parameters ϵ that are independent of x as well, projecting the rest out of the resulting $N = 1$, $d = 9$ theory which is the case $n = 1$ of Ref. [10]. If the Killing spinor of the ten-dimensional configuration depends on x, the configuration will not be supersymmetric in nine dimensions. The effect of the nine-dimensional hidden symmetries in the ten-dimensional supersymmetry is unknown, while

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[1] It was suggested to us by Tseytlin to check whether supersymmetry is preserved by duality in this case.

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in the nine-dimensional theory is just an O(1,1) group completely consistent with supersymmetry [10]. This is exactly what happens in the counterexample above: the Killing spinor depends on φ when a φ-independent frame is used.

Recently Bakas [11] has found a more interesting example of the loss of unbroken supersymmetries after a series of T and S-duality transformations, T duality being the responsible of this loss. In his scheme supersymmetry is lost if the Killing vector with respect to which one dualizes has not self-dual covariant derivatives. We believe that his example also satisfies our criterion: if one calculated explicitly the Killing spinors of such configurations[2] in an x-independent frame, they would depend on the isometry direction x. We hope these different criteria can be shown to be equivalent for these configurations.

This work is structured as follows. In Sec. II we set up the general problem of dimensionally reducing one dimension in the low-energy string effective action in absence of gauge fields, mainly for fixing the conventions and notation. We describe the effect of T duality on the compactified dimension from the point of view of the lower-dimensional theory. In Sec. III we study the effect of T duality on the supersymmetry properties of purely bosonic configurations of the zero-slope limit of heterotic string theory in ten dimensions. Accordingly we investigate the behavior under T duality of the supersymmetry rules of pure $N=1$, $d=10$ supergravity. Using a Kaluza-Klein basis of zehnbeins we rewrite the ten-dimensional supersymmetry transformation rules in a manifestly T-duality-invariant form for configurations which have unbroken supersymmetries with the Killing spinor independent on the isometry direction. In Sec. IV we present examples of configurations with (broken) unbroken supersymmetry after duality in accordance with (dependence) independence of the Killing spinor on isometry direction. Section V contains our conclusions. Finally, the Appendix contains some additional results of our work: we dimensionally reduce $N=1$, $d=10$ supergravity to $d=4$ and study the truncation of the lower-dimensional theory consisting in setting to zero all the fields which are *matter* from the point of view of $N=4$, $d=4$ supergravity. The remaining fields are found to be duality invariant. Therefore, when a supersymmetric compactification is done and the resulting theory is truncated to pure supergravity, T duality in the compactified directions has no effect whatsoever on the theory.

II. FROM D TO $D-1$ DIMENSIONS

The D-dimensional action we start from is

$$S = \frac{1}{2}\int d^D x\sqrt{-\hat{g}}e^{-2\hat{\phi}}[-\hat{R}+4(\partial\hat{\phi})^2-\tfrac{3}{4}\hat{H}^2]\,, \qquad (1)$$

where the fields are the metric, the axion, and the dilaton $\{\hat{g}_{\hat{\mu}\hat{\nu}}, \hat{B}_{\hat{\mu}\hat{\nu}}, \hat{\phi}\}$ and our conventions are those of Ref. [2]. In particular the axion field strength $\hat{H}$ is given by

$$\hat{H}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \partial_{[\hat{\mu}}\hat{B}_{\hat{\nu}\hat{\rho}]}\,. \qquad (2)$$

All the D-dimensional entities carry a caret and the $(D-1)$-dimensional ones do not. Then the indices take the values

$$\hat{\mu} = (0,\ldots,D-2,D-1) = (\mu, D-1)\,. \qquad (3)$$

We call the coordinate $x^{D-1}\equiv x$. To distinguish between curved and flat indices when confusion may arise, we underline the curved ones ($\xi^{\underline{x}}$, for instance). Now we assume that the fields are independent of the coordinate x, i.e., there exists a Killing vector $\hat{k}^{\hat{\mu}}$ such that

$$\hat{k}^{\hat{\mu}}\partial_{\hat{\mu}} = \partial_{\underline{x}}\,. \qquad (4)$$

Then, in this coordinate system, the components of the Killing vector are

$$\hat{k}^{\hat{\mu}} = \delta^{\hat{\mu}\underline{x}}\,,\quad \hat{k}_{\hat{\mu}} = \hat{g}_{\hat{\mu}\underline{x}}\,,\quad \hat{k}^2 = \hat{k}^{\hat{\mu}}\hat{k}_{\hat{\mu}} = \hat{g}_{\underline{xx}}\,, \qquad (5)$$

and the metric can be rewritten as

$$ds^2 = \hat{k}^{-2}(\hat{k}_{\hat{\mu}}dx^{\hat{\mu}})^2 + (\hat{g}_{\mu\nu} - \hat{k}^{-2}\hat{k}_\mu\hat{k}_\nu)dx^\mu dx^\nu\,. \qquad (6)$$

The Killing vector can be either timelike or spacelike, but not null. We will keep our expressions valid for both cases because from the point of view of T duality both are equally interesting [12] and the compactification of a timelike coordinate is not usually considered in the literature because it gives rise to an inconsistent lower-dimensional theory. We consider here the lower-dimensional theory just as a tool.

The above action enjoys invariance under the following Buscher's [13] T-duality transformations

$$\begin{aligned}
\tilde{\hat{g}}_{\underline{xx}} &= 1/\hat{g}_{\underline{xx}}\,,\quad \tilde{\hat{B}}_{\underline{x}\mu} = \hat{g}_{\underline{x}\mu}/\hat{g}_{\underline{xx}}\,, \tilde{\hat{g}}_{\underline{x}\mu} = \hat{B}_{\underline{x}\mu}/\hat{g}_{\underline{xx}}\,,\\
\tilde{\hat{B}}_{\mu\nu} &= \hat{B}_{\mu\nu} + (\hat{g}_{\underline{x}\mu}\hat{B}_{\nu\underline{x}} - \hat{g}_{\underline{x}\nu}\hat{B}_{\mu\underline{x}})/\hat{g}_{\underline{xx}}\,, \qquad (7)\\
\tilde{\hat{g}}_{\mu\nu} &= \hat{g}_{\mu\nu} - (\hat{g}_{\underline{x}\mu}\hat{g}_{\underline{x}\nu} - \hat{B}_{\underline{x}\mu}\hat{B}_{\underline{x}\nu})/\hat{g}_{\underline{xx}},\\
\tilde{\hat{\phi}} &= \hat{\phi} - \frac{1}{2}\ln|\hat{g}_{\underline{xx}}|\,.
\end{aligned}$$

Checking directly the invariance of the action Eq. (1) under the above transformations is a very involved calculation but if we compactify the redundant coordinate x, checking duality will be very easy.

Now we are going to dimensionally reduce the above action to $D-1$ dimensions by compactifying the redundant dimension x. We use the standard techniques of Scherk and Schwarz [14]. First we parametrize the D-bein as follows

$$(\hat{e}_{\hat{\mu}}{}^{\hat{a}}) = \begin{pmatrix} e_\mu{}^a & kA_\mu \\ 0 & k\end{pmatrix}\,,\quad (\hat{e}_{\hat{a}}{}^{\hat{\mu}}) = \begin{pmatrix} e_a{}^\mu & -A_a \\ 0 & k^{-1}\end{pmatrix}\,, \qquad (8)$$

where

$$k = |\hat{k}_{\hat{\mu}}\hat{k}^{\hat{\mu}}|^{\frac{1}{2}}\,, \qquad (9)$$

[2] To apply our criterion one has to find the Killing spinors explicitly.

and $A_a = e_a{}^\mu A_\mu$. The functions $\hat{e}_{\hat{\mu}}{}^{\hat{a}}$ do not depend on x. Note that $\hat{k}_{\hat{\mu}}\hat{k}^{\hat{\mu}} = \hat{\eta}_{xx}k^2$. With our conventions (mostly minuses signature) $\hat{\eta}_{xx}$ is positive if x is a timelike coordinate and $\hat{k}^{\hat{\mu}}$ a timelike vector, and $\hat{\eta}_{xx}$ is negative if x and $\hat{k}^{\hat{\mu}}$ are both spacelike.

With this parametrization, the D-dimensional fields decompose as follows:

$$\begin{aligned}
&\hat{g}_{\underline{xx}} = \hat{\eta}_{xx}k^2, \quad \hat{B}_{\underline{x}\mu} = B_\mu\,, \\
&\hat{g}_{\underline{x}\mu} = \hat{\eta}_{xx}k^2 A_\mu\,, \quad \hat{B}_{\mu\nu} = B_{\mu\nu} + A_{[\mu}B_{\nu]}\,, \\
&\hat{g}_{\mu\nu} = g_{\mu\nu} + \hat{\eta}_{xx}k^2 A_\mu A_\nu\,, \quad \hat{\phi} = \phi + \frac{1}{2}\ln k\,,
\end{aligned} \tag{10}$$

where $\{g_{\mu\nu}, B_{\mu\nu}, A_\mu, B_\mu, k, \phi\}$ are the $(D-1)$-dimensional fields. They are given in terms of the D-dimensional fields by

$$\begin{aligned}
&g_{\mu\nu} = \hat{g}_{\mu\nu} - \hat{g}_{\underline{x}\mu}\hat{g}_{\underline{x}\nu}/\hat{g}_{\underline{xx}}\,, \quad B_\mu = \hat{B}_{\underline{x}\mu}\,, \\
&B_{\mu\nu} = \hat{B}_{\mu\nu} + \hat{g}_{\underline{x}[\mu}\hat{B}_{\nu]\underline{x}}/\hat{g}_{\underline{xx}}\,, \quad \phi = \hat{\phi} - \frac{1}{4}\ln|\hat{g}_{\underline{xx}}|\,, \\
&A_\mu = \hat{g}_{\underline{x}\mu}/\hat{g}_{\underline{xx}}\,, \quad k = |\hat{g}_{\underline{xx}}|^{\frac{1}{2}}\,.
\end{aligned} \tag{11}$$

Then the D-dimensional action Eq. (1) is identically equal to

$$\begin{aligned}
S = \frac{1}{2}\int d^{(D-1)}x\sqrt{\eta_{xx}g}\,e^{-2\phi}[-R + 4(\partial\phi)^2 - \tfrac{3}{4}H^2 \\
-(\partial \ln k)^2 - \hat{\eta}_{xx}\tfrac{1}{4}k^2F^2(A) - \hat{\eta}_{xx}\tfrac{1}{4}k^{-2}F^2(B)]\,,
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
&F_{\mu\nu}(A) = 2\partial_{[\mu}A_{\nu]}\,, \quad F_{\mu\nu}(B) = 2\partial_{[\mu}B_{\nu]}\,, \\
&H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} + \frac{1}{2}A_{[\mu}F_{\nu\rho]}(B) + \frac{1}{2}B_{[\mu}F_{\nu\rho]}(A)\,,
\end{aligned} \tag{13}$$

are the vectors and antisymmetric tensor field strengths.

Equation (12) can be interpreted as a $(D-1)$-dimensional action for the above $(D-1)$-dimensional fields. Observe that, when x is a timelike coordinate, the vector fields kinetic terms have the wrong signs in the above action.

Now, using first the definitions of the $(D-1)$-dimensional fields in terms of the D-dimensional ones, Eqs. (11), and Buscher's duality rules, Eqs. (7), it is very easy to check that the duals of the $(D-1)$-dimensional fields are

$$\begin{aligned}
&\tilde{g}_{\mu\nu} = g_{\mu\nu}\,, \quad \tilde{A}_\mu = B_\mu\,, \\
&\tilde{B}_{\mu\nu} = B_{\mu\nu}\,, \quad \tilde{B}_\mu = A_\mu\,, \\
&\tilde{\phi} = \phi\,, \quad \tilde{k} = k^{-1}\,,
\end{aligned} \tag{14}$$

that is, in the $(D-1)$-dimensional theory the only effect of T duality is to interchange the vector fields A_μ and B_μ and to invert k. This is an obvious symmetry of the $(D-1)$-dimensional action, Eq. (12), which, on the other hand is identically equal to the D-dimensional one, Eq. (1). From the lower-dimensional point of view, the invariance of the action under T duality is manifest.

Observe that, in particular, the $(D-1)$-beins are duality invariant. This is completely consistent with the transformation rules derived in Ref. [2]

$$\begin{aligned}
&\tilde{\hat{e}}_{\underline{x}}{}^{\hat{a}} = \frac{1}{\hat{g}_{\underline{xx}}}\hat{e}_{\underline{x}}{}^{\hat{a}}\,, \\
&\tilde{\hat{e}}_\mu{}^{\hat{a}} = \mp\hat{e}_\mu{}^{\hat{a}} - \frac{1}{\hat{g}_{\underline{xx}}}(\hat{g}_{\underline{x}\mu} \pm \hat{B}_{\underline{x}\mu})\hat{e}_{\underline{x}}{}^{\hat{a}}\,,
\end{aligned} \tag{15}$$

and it is this property of the Kaluza-Klein basis, Eq. (8), which simplifies the transformation rules is justifies its use here.

Of course, the transformation one sees in the lower-dimensional theory is part of the $\mathrm{O}(d,d)$ symmetry exhibited in Ref. [7] when one compactifies d dimensions. Now we have made this relation very explicit and it is going to be extremely useful for the study of the unbroken supersymmetries of the dual configurations.

III. DUALITY VERSUS SUPERSYMMETRY

In this section we investigate the general relation between unbroken supersymmetries before and after a T-duality transformation using the results of the previous section with $D = 10$. Specifically we are going to analyze the effect of a T-duality transformation on $N = 1$, $d = 10$ supergravity Killing spinors. To do this one needs to know how the zehnbeins transform under duality. As we explained in the previous section, the zehnbein duality transformation laws were found in Ref. [2] and reduce to Eqs. (14) for the x-independent Kaluza-Klein basis of zehnbeins, Eq. (8), where a clear distinction between the cases in which unbroken supersymmetry is preserved and those in which it is not arises naturally.

We consider here the zero slope limit of the heterotic string theory without gauge fields, which is given by $N = 1$, $d = 10$ supergravity. The bosonic part of the action of $N = 1$, $d = 10$ supergravity in absence of vector fields is given by Eq. (1) with $D = 10$:

$$S = \frac{1}{2}\int d^{10}x\sqrt{-\hat{g}}\,e^{-2\hat{\phi}}[-\hat{R} + 4(\partial\hat{\phi})^2 - \tfrac{3}{4}\hat{H}^2]\,, \tag{16}$$

with H given by Eq. (2). The corresponding fermionic supersymmetry transformation rules are

$$\begin{aligned}
&\delta_{\hat{\epsilon}}\hat{\psi}_{\hat{c}} = [\partial_{\hat{c}} - \tfrac{1}{4}(\hat{\omega}_{\hat{c}}{}^{\hat{a}\hat{b}} - \tfrac{3}{2}\hat{H}_{\hat{c}}{}^{\hat{a}\hat{b}})\hat{\gamma}_{\hat{a}\hat{b}}]\hat{\epsilon}\,, \\
&\delta_{\hat{\epsilon}}\hat{\lambda} = (\hat{\gamma}^{\hat{c}}\partial_{\hat{c}}\hat{\phi} + \tfrac{1}{4}\hat{H}_{\hat{a}\hat{b}\hat{c}}\hat{\gamma}^{\hat{a}\hat{b}\hat{c}})\hat{\epsilon}\,.
\end{aligned} \tag{17}$$

Now we assume that some spinor $\hat{\epsilon}$ makes these equations vanish (i.e., $\hat{\epsilon}$ is a Killing spinor[3]) for some specific x-independent field configuration and we want to investigate whether this $\hat{\epsilon}$ is also a Killing spinor of the T-dual field configuration or whether it is related to another Killing spinor of the dual field configuration, as in the S-duality case [15]. To investigate this problem

[3]Actually a spinor that makes Eqs. (17) vanish needs to have a specific asymptotic behavior in order to be a Killing spinor, but these details will not concern us in this discussion.

we rewrite the above equations in terms of the nine-dimensional fields. It is perhaps worth stressing that we are not reducing the ten-dimensional γ matrices nor the ten-dimensional spinors, which are the objects we are interested in. Again, here, dimensional reduction can be understood as a tool for having under control the duality transformations. All the indices used below are flat. We also use the slightly unusual notation (observe that the γ matrices are ten-dimensional but the indices contracted are the nine-dimensional ones)

$$\not{\partial} \equiv \hat{\gamma}^a \partial_a\,, \quad \not{F} \equiv \hat{\gamma}^{ab} F_{ab}\,, \quad \not{F}_a \equiv \hat{\gamma}^b F_{ab}\,, \quad \not{H} \equiv \hat{\gamma}^{abc} H_{abc}\,. \tag{18}$$

We get for the x component (flat) and the a component of the gravitino transformation and for the dilatino transformation

$$\begin{aligned}
&\hat{\gamma}_x \delta_{\hat{\epsilon}} \hat{\psi}_x = \{k^{-1}\partial_{\underline{x}} - \tfrac{1}{8}[\hat{\eta}_{xx} k \not{F}(A) + k^{-1} \not{F}(B)] + \tfrac{1}{2}\hat{\gamma}_x(\not{\partial} \ln k)\}\hat{\epsilon} = 0\,,\\
&\delta_{\hat{\epsilon}} \hat{\psi}_a = \{[\partial_a - \tfrac{1}{4}(\omega_a{}^{bc} - \tfrac{3}{2} H_a{}^{bc})\hat{\gamma}_{bc}] - \tfrac{1}{8}\hat{\gamma}_x[\not{F}_a(A) - \hat{\eta}_{xx} \not{F}_a(B)] - A_a \partial_{\underline{x}}\}\hat{\epsilon} = 0\,,\\
&\delta_{\hat{\epsilon}} \hat{\lambda} = \{\not{\partial}\phi + \tfrac{1}{4} \not{H} - \tfrac{1}{4}\hat{\eta}_{xx} k^{-1} \hat{\gamma}_x \not{F}(B) + \tfrac{1}{2} \not{\partial} \ln k\}\hat{\epsilon} = 0\,,
\end{aligned} \tag{19}$$

respectively.

As they stand, none of these equations is separately manifestly duality invariant. Unless we assume in what follows that the Killing spinor of the original configuration does not depend on the isometry direction x, no further progress can be made in relating the supersymmetry of the original configurations with that of the final one. Thus we require that

$$\partial_{\underline{x}} \hat{\epsilon} = 0\,. \tag{20}$$

Using this assumption we have the Killing spinor equations in the form

$$\begin{aligned}
&\{\tfrac{1}{8}[\hat{\eta}_{xx} k \not{F}(A) + k^{-1} \not{F}(B)] - \tfrac{1}{2}\hat{\gamma}_x(\not{\partial} \ln k)\}\hat{\epsilon} = 0\,,\\
&\{[\partial_a - \tfrac{1}{4}(\omega_a{}^{bc} - \tfrac{3}{2} H_a{}^{bc})\hat{\gamma}_{bc}] - \tfrac{1}{8}\hat{\gamma}_x[\not{F}_a(A) - \hat{\eta}_{xx} \not{F}_a(B)]\}\hat{\epsilon} = 0\,,\\
&\{\not{\partial}\phi + \tfrac{1}{4} \not{H} - \tfrac{1}{4}\hat{\eta}_{xx} k^{-1} \hat{\gamma}_x \not{F}(B) + \tfrac{1}{2} \not{\partial} \ln k\}\hat{\epsilon} = 0\,.
\end{aligned} \tag{21}$$

Still, after assuming $\partial_{\underline{x}}\hat{\epsilon} = 0$, not all of the Killing spinor equations are manifestly separately duality invariant. To be precise (and here we take $\hat{\eta}_{xx} = -1$) using the nine-dimensional version of Buscher's duality rules, Eq. (14), the first and the second are duality invariant but the third is clearly not. However, since by assumption all of them are satisfied by $\hat{\epsilon}$, we are allowed to combine them. If we substitute the first into the third, we get the duality-invariant equation

$$\{\not{\partial}\phi + \tfrac{1}{4} \not{H} + \tfrac{1}{8}\hat{\gamma}_x[k \not{F}(A) + k^{-1} \not{F}(B)]\}\hat{\epsilon} = 0\,. \tag{22}$$

This proves that $\hat{\epsilon}$ is a Killing spinor of the dual configuration if it is so for the original configuration, that is

$$\tilde{\hat{\epsilon}} = \hat{\epsilon}\,. \tag{23}$$

If we take $\hat{\eta}_{xx} = +1$ (timelike duality) it is easy to see that

$$\tilde{\hat{\epsilon}} = \hat{\gamma}_x \hat{\epsilon}\,. \tag{24}$$

Examples of these results will be discussed in Sec. III.

The set of T-duality-invariant supersymmetry equations that we have generated by dimensional reduction should be nothing but the explicitly O(1,1)-invariant $N = 1$, $d = 9$ supergravity theory of Ref. [10] for the case $n = 1$ and in stringy frame, although the dimensional reduction of the supersymmetry parameters, γ matrices, etc., still has to be done. There are factors of e^{ϕ} relating the Einstein-frame and string-frame spinors too and the comparison between our results and those or Ref. [10] is not straightforward. It is clear, however, that the correspondence disappears if we do not impose Eq. (20) to the supersymmetry parameters.

We would like to stress that we have derived the condition of preservation of unbroken supersymmetry Eq. (20) using heavily a zehnbein basis of the form Eq. (8). However, after deriving this condition in that special frame we may ask ourselves to which extent this condition is frame dependent. The answer is that the same criterion is valid in any x-independent frame. Indeed, if one changed from the x-independent Kaluza-Klein frame discussed above to any other x-independent frame, the Lorentz rotation involved would not change the fact that the spinor is or is not x dependent since the same x-independent parameter $\hat{\omega}^{\hat{a}\hat{b}}$ appears in the spinors and frames transformation laws $\hat{\epsilon}' = \exp(\frac{1}{4}\hat{\omega}^{\hat{a}\hat{b}}\hat{\gamma}_{\hat{a}\hat{b}})\hat{\epsilon}$ and $\hat{e}_{\hat{\mu}} = \exp(\frac{1}{4}\hat{\omega}^{\hat{a}\hat{b}}\hat{M}_{\hat{a}\hat{b}})\hat{e}_{\hat{\mu}}$, where the $\hat{M}_{\hat{a}\hat{b}}$s are the generators of the ten-dimensional Lorentz group in the vector representation.

IV. EXAMPLES

In this section we are going to study examples of supersymmetric configurations and duality transformations which illustrate the results of Sec. III.

(1) *Losers*: configurations that lose their unbroken supersymmetries after T duality.

(i) Our first example is flat ten-dimensional space-time in polar coordinates $\rho^2 = (x^1)^2 + (x^2)^2$, $\tan\varphi = x^2/x^1$:

$$ds^2 = dt^2 - d\rho^2 - \rho^2 d\varphi^2 - dx^I dx^I\,, \quad I = 3, \ldots, 9\,. \tag{25}$$

This solution of $N = 1$, $d = 10$ supergravity has all supersymmetries unbroken. In the zehnbein basis

$$e_t{}^0 = 1, \quad e_\rho{}^1 = 1, \quad e_\varphi{}^2 = \rho, \quad e_{\underline{I}}{}^J = \delta_{\underline{I}}{}^J, \tag{26}$$

which is of the type of that in Eq. (8), the Killing spinors are given by

$$\epsilon = e^{-\frac{1}{4}\gamma_1\gamma_2\varphi}\epsilon_0, \tag{27}$$

where ϵ_0 is a completely arbitrary constant spinor.[4]

After a duality transformation in the direction φ, we get the solution

$$ds^2 = dt^2 - d\rho^2 - \rho^{-2}d\varphi^2 - dx^I dx^I, \quad \phi = -\ln\rho. \tag{28}$$

The dilatino supersymmetry rule implies that the Killing spinors of this solution have to satisfy the constraint

$$\gamma^1\epsilon = 0, \tag{29}$$

which can only be satisfied by $\epsilon = 0$. Therefore, all the supersymmetries of the dual solution of Minkowski space are broken. As we saw in Sec. III this is related to the dependence of the Killing spinors on φ when we use adapted coordinates and a φ-independent frame.

(ii) Our second example is the one recently found by Bakas in Ref. [11]. He studied self-dual Euclidean metrics admitting a Killing vector associated to the coordinate τ, which generally can be written in the form

$$ds^2 = V(d\tau + \omega_i dx^i)^2 + V^{-1}\gamma_{ij}dx^i dx^j. \tag{30}$$

Self-duality of the metric is an integrability condition for the existence of unbroken supersymmetries. What was actually observed in [11] was the breaking of the self-duality condition of the configuration after the T-S-T chain of duality transformations.

The violation of supersymmetry in this example could be attributed to T-duality, since, as we have said, S-duality is perfectly consistent with supersymmetry. Furthermore, the violation of supersymmetry by T duality was related to the nature of the Killing vector: T duality with respect to "translational" Killing vectors would not violate supersymmetry while T duality with respect to "rotational" Killing vectors would. In particular, this criterion was sufficient to show that for configurations with flat three-dimensional metrics $\gamma_{ij} = \delta_{ij}$ no violation of supersymmetry happened. However, for some special choices of nonflat γ_{ij} the self-duality of the final configuration was violated.

From our point of view, this gives an interesting example of our general statement that unless the Killing spinor in Kaluza-Klein basis is shown to be independent on duality direction there is no reason to expect the preservation of supersymmetry by T duality. A preliminary study shows that all the cases found in Ref. [11] to violate supersymmetry suffer from the problem of dependence of the Killing spinor on the coordinate associated with the isometry. Observe that one of his examples with $V = 1$ and $\gamma_{ij} \neq \delta_{ij}$ is provided by case (i) above.

(2) *Winners*: configurations with unbroken supersymmetries that are preserved by T duality. Alternatively we could refer to them as those configurations with unbroken supersymmetries and x-independent Killing spinors since the results of Sec. III guarantee, without the need of further proof, the supersymmetry of the dual configurations.

(i) The first example is provided by the SSW solutions and the generalized fundamental string (GFS) solutions which are both supersymmetric and are known to be related by duality in the direction x [1–3]. Let us describe briefly these two classes of solutions. The SSW solutions are

$$\begin{aligned}
\hat{ds}^2 &= 2dudv + 2\mathcal{A}_u du^2 + 2\mathcal{A}_{\underline{i}}dx^{\underline{i}}du - dx^{\underline{i}}dx^{\underline{i}}, \\
\hat{B} &= 2\mathcal{A}_{\underline{i}}dx^{\underline{i}} \wedge du, \\
\hat{\phi} &= 0,
\end{aligned} \tag{31}$$

and the GFS solutions are[5]

$$\begin{aligned}
\hat{ds}^2 &= 2e^{2\hat{\phi}}\{dudv + \mathcal{A}_{\underline{i}}dx^{\underline{i}}du\} - dx^{\underline{i}}dx^{\underline{i}}, \\
\hat{B} &= -2e^{2\hat{\phi}}\{(1 - e^{-2\hat{\phi}})du \wedge dv + \mathcal{A}_{\underline{i}}du \wedge dx^{\underline{i}}\}, \\
\hat{\phi} &= -\frac{1}{2}\ln(1 - \mathcal{A}_u).
\end{aligned} \tag{32}$$

Here $i = 1, \ldots, 8$, $u = \frac{1}{\sqrt{2}}(t + x)$, $v = \frac{1}{\sqrt{2}}(t - x)$, and the fields do not depend on $x = x^9$ and on $t = x^0$.

To use the machinery developed in the main body of the paper we have to identify the nine-dimensional fields. For our purposes it is enough to do it for just the SSW solutions. First of all we need a zehnbein basis of the form of Eq. (8). Fields k and A_μ that appear in it are readily identified:

$$k = (1 - \mathcal{A}_u)^{\frac{1}{2}}, \quad A_t = \frac{k^2 - 1}{k^2}, \quad A_{\underline{i}} = -\frac{1}{\sqrt{2}k^2}\mathcal{A}_{\underline{i}}. \tag{33}$$

A *neunbein* basis, necessary to complete the zehnbein basis, is provided by

$$(e_\mu{}^a) = \begin{pmatrix} k^{-1} & 0 \\ -kA_{\underline{i}} & \delta_{\underline{i}}{}^j \end{pmatrix}, \quad (e_a{}^\mu) = \begin{pmatrix} k & 0 \\ k^2 A_{\underline{i}} & \delta_i{}^{\underline{j}} \end{pmatrix}, \tag{34}$$

and the rest of the nine-dimensional fields are (with curved indices)

$$\begin{aligned}
B_{t\underline{i}} &= -\frac{1}{2\sqrt{2}}\frac{1 + k^2}{k^2}, \quad B_t = 0, \\
B_{\underline{ij}} &= 0, \quad B_{\underline{i}} = -\frac{1}{\sqrt{2}}\mathcal{A}_{\underline{i}} = k^2 A_{\underline{i}}, \\
\phi &= 0.
\end{aligned} \tag{35}$$

[4]In Cartesian coordinates and in the most obvious frame $\hat{e}_{\hat{\mu}}{}^{\hat{a}} = \delta_{\hat{\mu}}{}^{\hat{a}}$ the Killing spinors are just arbitrary constant spinors and so have the right asymptotic behavior.

[5]In order to avoid ambiguities we will always assume that $\mathcal{A}_u - 1 < 0$ so the solution will always have the same signature as the asymptotic infinity (when the fields vanish).

The field strengths of the nine-dimensional vector fields A, B are given by

$$F_{0i}(A) = -2k^{-2}\partial_{\underline{i}}k\,,\quad F_{0i}(B) = 0\,,$$
$$F_{ij}(A) = -4k^{-1}A_{[\underline{i}}\partial_{\underline{j}]}k + F_{\underline{ij}}(A)\,,\quad F_{ij}(B) = k^2 F_{ij}(A)\,. \tag{36}$$

If we write the Killing spinor equation $\delta_\epsilon\hat{\psi}_{\underline{x}} = 0$ in the form

$$(\partial_{\underline{x}} + M)\hat{\epsilon} = 0\,, \tag{37}$$

we have

$$M = \frac{1}{8}\{k^2\ \not{F}(A) - \not{F}(B) - 4\hat{\gamma}^x\ \not{\partial}k\} = \frac{1}{2}(\hat{\gamma}^0 - \hat{\gamma}^x)(\hat{\gamma}^i\partial_{\underline{i}}k)\,. \tag{38}$$

This implies that the Killing spinor is x independent $\partial_{\underline{x}}\hat{\epsilon}$ if it is constrained by

$$(\hat{\gamma}^0 - \hat{\gamma}^x)\hat{\epsilon} = 0\,. \tag{39}$$

This is just the constraint found in Ref. [1], using a different (but also x-independent) zehnbein basis, though. As it was explained in the end of the previous section, the independence of the Killing spinor on x in a basis of the form of Eqs. (8) and (34) follows from its x independence on any other x-independent frame, in particular that of Ref. [1].

(ii) A second example is provided by the dual relation between a special class of fivebrane solutions [4] called multimonopoles in Ref. [5] and the stringy ALE instantons [6] which have the multicenter Gibbons-Hawking metric. It was observed in Ref. [6] that these two solutions are related by T duality. The reason why only the multimonopole configurations are dual to the stringy ALE instantons is simple. The characteristic property of those class of fivebranes is the independence on the direction $x^4 = \tau$ which is the one used for duality. Generic fivebrane [4] as well as generic self-dual metrics [6] do not have such an isometry. The fivebrane solutions, including the multimonopoles, have unbroken supersymmetries with constant chiral (in four-dimensional Euclidean space) Killing spinors in a τ-independent zehnbein basis. According to the results of the previous section this would be sufficient to claim that the dual solutions (the stringy ALE instantons) have unbroken supersymmetries with the same Killing spinors.

(iii) Our last example illustrates our results for timelike duality, although it cannot be said it is a natural born "winner." It is easy to show that the extreme magnetic dilaton black hole, uplifted to ten dimensions in [16], is invariant under timelike duality. We also know that it has unbroken supersymmetries with constant Killing spinors restricted by the same condition as the fivebrane Killing spinors of Ref. [4]: the Killing spinors are chiral in the four-dimensional Euclidean space spanned by the coordinates $x^1, \ldots, x^4$, that is

$$(1 \pm \hat{\gamma}_{1234})\hat{\epsilon}_\pm = 0\,. \tag{40}$$

Since the configuration is invariant, the Killing spinors *are* invariant too. On the other hand, in Sec. III we found that the Killing spinors change after timelike duality according to Eq. (24). There seems to be a contradiction between these two facts, but, actually, they are consistent with each other because the above constraint is invariant under multiplication by $\hat{\gamma}_0 (\equiv \hat{\gamma}_x)$ and the Killing spinor is simply transformed into another Killing spinor.

It would be interesting to apply our results to supersymmetric configurations which are not timelike invariant since in general timelike duality seems to change the sign of the energy of the configurations and interchanges singularities and horizons [12, 17] while supersymmetry (as we have shown) is preserved.

V. CONCLUSION

Bosonic configurations may have Killing vectors and, when embedded in a supergravity theory, also Killing spinors. We have studied the case in which both are present and one performs a T duality transformation in the direction associated to a Killing vector.

Usually, the existence of a Killing vector means that there exist a system of coordinates (adapted coordinates) in which the fields [here the metric (or zehnbeins), the dilaton, and the two-form field] do not depend on the coordinate associated to the Killing vector. One of our main conclusions is that if a bosonic configuration admits a Killing vector and a Killing spinor and one uses adapted coordinates, even if the bosonic fields do not depend on the coordinate associated to the isometry it is not guaranteed that the Killing spinor will not depend on it as well. We have exhibited different examples of this situation. Our second main conclusion is that in this situation, if one performs a T-duality transformation in the direction associated to the Killing vector, the dual configuration will not admit Killing spinors.

The main result of our paper is that T duality does preserve the unbroken supersymmetries of those configurations whose Killing spinors are independent of the coordinate associated with the isometry used for duality and the Killing spinors transform in a very simple way.

It is interesting to compare this situation with the case of S duality. S duality always preserves the unbroken supersymmetries of the configurations at the classical level [15]. However S duality and T duality are on equal footing in some contexts [18]: when the effective action of the type-II superstring is compactified on a six torus, the hidden symmetry of the resulting four-dimensional theory ($N = 8$ supergravity) is E_7, which contains the SO(6,6) T-duality group and the $SL(2, R)$ S-duality group. Obviously, from the four-dimensional point of view, both T and S duality must be consistent with supersymmetry. However, in the case of T duality, we are not interested in four-dimensional configurations for which T duality amounts to a rotation of vector and scalar fields but, often, we are interested in the nontrivial effects induced by T duality in the ten-dimensional metric. From the ten-dimensional point of view (the one we adopt here) T duality will not be consistent with supersymmetry in the cases explained above.

The investigation of α' corrections with respect to T duality may also lead to the discovery of some new features. We know that T duality gets α' corrections and this means that the hidden symmetries of the conventional supergravity theories (and the theories themselves) will be modified in a form unknown at present time. We have some relevant results on T duality which includes non-Abelian vector fields and α' corrections which explain the fact that the SSW [1] solutions as well as the dual wave solutions [2] have unbroken supersymmetry with account of α' corrections. These results will be published elsewhere [8].

Note added in proof. Note that for $\rho = 0$ the $\varphi\varphi$ component of the metric given in Eq. (25) vanishes. This leads to a singular point in the dual metric given in Eq. (28). Nevertheless, one can perform a duality transformation in the φ direction as has been discussed in Ref. [23].

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APPENDIX: FROM $d = 10$ TO $d = 4$: THE SUPERSYMMETRIC TRUNCATION

Now we want to make connection with the action of $N = 4$, $d = 4$ supergravity. Therefore we have to compactify six spacelike coordinates and we substitute everywhere $\hat{\eta}_{xx} = -1$. The compactification from $N = 1$, $d = 10$ to $N = 4$, $d = 4$ was done by Chamseddine in Ref. [19]. However Chamseddine worked in the Einstein frame and that makes it very difficult to study the effect of T duality on the resultant theory. Our goal here will be to obtain pure $N = 4$, $d = 4$ supergravity (or part of it) in string frame, identifying which fields belong to the matter multiplet and which fields belong to the supergravity multiplet and how the dimensionally reduced action has to be truncated in order to get rid of the matter fields.

We perform the dimensional reduction of the theory from $d = 10$ to $d = 4$ for a simplified model in which most of the $d = 10$ fields are trivial. This simplified model is enough to discuss the important features of the dimensional reduction versus duality.

We do it in two steps. First we reduce from $d = 10$ to $D = 5$. We denote the ten-dimensional fields by an upper index 10 and the five-dimensional fields by a caret. The ten-dimensional indices are capital letters $M, N = 0, \ldots, 9$, the five-dimensional indices will carry a caret $\hat{\mu}, \hat{\nu} = 0, \ldots, 4$, and the compactified dimensions will be denoted by capital I's and J's, $I, J = 5, \ldots, 9$. We take the $d = 10$ fields to be related to the $D = 5$ ones by

$$\begin{aligned} &g^{(10)}_{\hat{\mu}\hat{\nu}} = \hat{g}_{\hat{\mu}\hat{\nu}}\,, \quad B^{(10)}_{\hat{\mu}\hat{\nu}} = \hat{B}_{\hat{\mu}\hat{\nu}}\,,\\ &g^{(10)}_{I\hat{\nu}} = 0\,, \quad B^{(10)}_{I\hat{\nu}} = 0\,,\\ &g^{(10)}_{IJ} = \eta_{IJ} = -\delta_{IJ}\,, \quad B^{10}_{IJ} = 0\,,\\ &\phi^{(10)} = \hat{\phi}\,. \end{aligned} \tag{A1}$$

We get

$$S = \frac{1}{2}\int d^5x\sqrt{-\hat{g}}e^{-2\hat{\phi}}[-\hat{R} + 4(\partial\hat{\phi})^2 - \tfrac{3}{4}\hat{H}^2]\,. \tag{A2}$$

As a second step we reduce from $D = 5$ to $d = D - 1 = 4$ using the results and notation of the previous section. We get

$$\begin{aligned} S = \frac{1}{2}\int d^4x e^{-2\phi}\sqrt{-g}[&-R + 4(\partial\phi)^2 - \tfrac{3}{4}H^2\\ &-(\partial\ln k)^2 + \tfrac{1}{4}k^2F^2(A) + \tfrac{1}{4}k^{-2}F^2(B)]\,. \end{aligned} \tag{A3}$$

Now, if we look to the gravitino supersymmetry rule in $d = 10$,

$$\delta_\epsilon\hat{\psi}_{\hat{c}} = [\partial_{\hat{c}} - \tfrac{1}{4}(\hat{\omega}_{\hat{c}}{}^{\hat{a}\hat{b}} - \tfrac{3}{2}\hat{H}_{\hat{c}}{}^{\hat{a}\hat{b}})\hat{\gamma}_{\hat{a}\hat{b}}]\epsilon\,, \tag{A4}$$

and observe that, setting $k = 1$

$$\hat{\omega}_c{}^{a4} - \frac{3}{2}\hat{H}_c{}^{a4} = -\frac{1}{2}F_c{}^a(A + B)\,, \tag{A5}$$

it is clear that the identification of the matter vector fields D_μ and the supergravity vector fields V_μ is the same as in Chamseddine's paper up to factors of 1/2:

$$\begin{aligned} D_\mu &= \frac{1}{2}(A_\mu - B_\mu)\,,\\ V_\mu &= \frac{1}{2}(A_\mu + B_\mu)\,, \end{aligned} \tag{A6}$$

respectively. We also have to put $k = 1$, because there is no such a scalar in the $N = 4$, $d = 4$ supergravity multiplet. Now we want to truncate the theory keeping only the supergravity vector field V_μ. We have then

$$k = 1\,, \quad V_\mu = A_\mu = B_\mu\,, \quad D_\mu = 0\,. \tag{A7}$$

The truncated action is

$$S = \frac{1}{2}\int d^4x\sqrt{-g}e^{-2\phi}[-R + 4(\partial\phi)^2 - \tfrac{3}{4}H^2 + \tfrac{1}{2}F^2(V)]\,, \tag{A8}$$

where

$$\begin{aligned} F_{\mu\nu}(V) &= 2\partial_{[\mu}V_{\nu]}\,,\\ H_{\mu\nu\rho} &= \partial_{[\mu}B_{\nu\rho]} + V_{[\mu}F_{\nu\rho]}(V)\,. \end{aligned} \tag{A9}$$

The embedding of the four-dimensional fields in this action in $d = 10$ is

$$\begin{aligned} &g^{(10)}_{\mu\nu} = g_{\mu\nu} - V_\mu V_\nu\,, \quad B^{(10)}_{\mu\nu} = B_{\mu\nu}\,,\\ &g^{(10)}_{\underline{x}\nu} = -V_\nu\,, \quad B^{(10)}_{\underline{x}\nu} = V_\nu\,,\\ &g^{(10)}_{\underline{xx}} = -1\,, \quad \phi^{(10)} = \phi\,,\\ &g^{(10)}_{IJ} = \eta_{IJ} = -\delta_{IJ}. \end{aligned} \tag{A10}$$

These formulas can be used to uplift any four-dimensional field configuration with one graviton, one axion, one vector, and a dilaton to a ten-dimensional field configuration in a way consistent with supersymmetry, as in Refs. [20, 16].

One obvious but important observation is that this action is not just invariant under x duality (here $x = x^4$), but *all* the fields that appear in it are individually invariant.[6]

But there is more. If we rewrite the truncation, Eq. (A7), in terms of the original ten-dimensional fields, it looks like this:

$$\begin{aligned} g^{(10)}_{\underline{xx}} &= -1\,,\\ g^{(10)}_{\underline{x}\mu} &= -B^{(10)}_{\underline{x}\mu}\,,\\ g^{(10)}_{\underline{x}I} &= B^{(10)}_{\underline{x}I} = 0\,. \end{aligned} \tag{A11}$$

Now one can check that a ten-dimensional configuration satisfying Eq. (A11) is invariant under T duality in the direction x. That is also obviously true for the rest of the compactified directions.

We can state this result as follows: if in the four-dimensional action we interpret the vector field V_μ as belonging to the supergravity multiplet[7] coming from the combination $A_\mu + B_\mu$, then the lifting to ten dimensions of any four-dimensional configuration will be an x duality invariant configuration if x is one of the compact dimensions.

One example is provided by the SSW [1] and the GFS [3, 2] solutions. These solutions are described in Sec. IV, Eqs. (31) and (32). If $x = x^9$ is the nontrivial compactified dimension (what we called before x^4), then, imposing the conditions Eq. (A11) means for these solutions $\mathcal{A}_u = \hat{\phi} = 0$. This subset of SSW and GFS are identical, are duality invariant in the $x = x^9$ direction and give rise to the same supersymmetric solutions of $N = 4$, $d = 4$ supergravity [20, 16, 21, 22].

[6] Note that the truncation itself is duality invariant, i.e., $\tilde{k} = k = 1$, $\tilde{D}_\mu = D_\mu = 0$.

[7] Which is necessary to have supersymmetry.

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